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ERROR ANALYSIS OF THE RECOVERY OF DATA **ACQUIRED BY A VOLTAGE-TO-FREQUENCY DATA** ACQUISITION SYSTEM FOR SINUSOIDAL, STEP, AND RAMP INPUTS

> G. L. Williams ARO. Inc.

August 1968

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G. L. Williams*
ARO, Inc.

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FOREWORD

The work reported herein was sponsored by the Arnold Engineering Development Center (AEDC), Air Force Systems Command (AFSC).

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This technical report has been reviewed and is approved.

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Directorate of Test

Roy R. Croy, Jr. Colonel, USAF Director of Test

ABSTRACT

The errors generated by the recovery of data acquired by a voltageto-frequency (V/F) data acquisition system are theoretically analyzed, using a digital computer. The input waveforms considered are the sinusoid, the step function, and the ramp function. In each case, the well known "sampling theorem" of C. E. Shannon, viz, two samples per period of the smallest period present in the signal are sufficient for recovery of data with perfect fidelity, will be shown to be inapplicable as a criterion for accurate and reliable recovery of data acquired by a V/F system. The errors are of two categories: One is the error from linear interpolation recovery, and the other is from the fact that data points do not always coincide with the input. The sinusoid analysis reveals that a sampling rate of 11.4 summing intervals per period is required to assure 5 percent or less error. The step function analysis shows that the errors are independent of sampling rate; however, its response time is approximately equal to the summing interval. The analysis of the ramp function indicated that the ratio of ramp rise time to summing interval must be five or greater to assure 5 percent or less error. These criteria are unique to each case, and in the case of the sinusoid, are many times larger than the criterion of the "sampling theorem. "

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	NOMENCLATURE	
E	Error at a data point	
е	Error at any point using linear interpolation	

f Frequency, Hz

Int () Integer part of

K Synchronization parameter for step and ramp input

K, Gain of V/F converter (pulses/sec-v)

m Summing intervals per period of sinusoid

N . Output counts of V/F converter in τ seconds

n Number of arbitrary summing interval

T Period of sinusoidal input, sec

T_D Time delay, sec

t Time

V Voltage

 α Argument of Eqs. (11) and (21)

 β Ratio of ramp rise time to summing interval

 Δ () Change in

 μ_1 Step function

 μ_2 Ramp function

au Length of summing interval, sec

 ψ Synchronization parameter for sinusoidal input

SUBSCRIPTS

i Input

max Maximum value min Minimum value

n Value in summing interval number n

o Output

SECTION I

Data acquisition relies heavily on the process of sampling and representing a continuous time function in terms of discrete time samples. Numerous methods and systems to accomplish this are in existence and have their own uniquenesses. However, all of these methods and systems have a common basis that justifies the representation of a continuous variable by discrete samples, and that is the sampling theorem.

The sampling theorem, though known and used much earlier, is widely attributed to C. E. Shannon (Ref. 1). It may be stated as follows:

"2/T samples per second suffice to represent perfectly and permit perfect recovery of a time function provided that the time function contains no periodic components of period less than T seconds."

However, the data recovery from an integrating system such as the voltage-to-frequency data acquisition system when used as a sampling system, i.e., used to recover the actual waveshape, does not have a direct application through the sampling theorem.

This investigation is a derivation of the number of integrating intervals required to accurately reconstruct an input using linear interpolation of the data points obtained from an integrating data acquisition system. Assuming that an error also exists at the data points, the analysis will include both a determination of data point error and linear interpolation error. Error will be the absolute value of the difference of the input and the representative output at a given time as a percentage of the maximum magnitude of the input. This is a modification of the definition of deviation as defined in Ref. 2, page 209.

Three ideal signals will be treated theoretically: sinusoid, step, and ramp. The digital computer will be used to simulate this type of data recovery, and the maximum error for various parameter configurations will be determined. From this, the limits of the parameters for accurate (5 percent) recovery can be determined.

Experimental verification of the error analysis for the sinusoid was performed using the VIDAR® voltage-to-frequency data acquisition system in the Large Rocket Facility at AEDC.

SECTION II VOLTAGE-TO-FREQUENCY CONVERSION METHOD

The method of data acquisition that is to be treated in this study is a voltage-to-frequency (V/F) conversion. The block diagram in Fig. 1 depicts the setup of such a system.

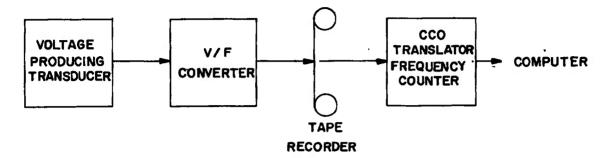


Fig. 1 Block Diagram of LRF V/F Acquistition and Recovery System

The V/F converter develops output pulses at a rate precisely proportional to the input voltage. The output pulses are recorded on magnetic tape for later data reduction. The recorded signals are recovered by means of a translator, which works like a gated frequency counter that counts the pulses during a time interval, τ . The counts contained in a summing interval are converted to a representative voltage, V_{τ_n} , by a digital computer using the calibration of the V/F converter and summing interval of the translator.

To illustrate: Let the V/F converter be adjusted so that the output is N_O counts in τ seconds with the input shorted. Then apply a calibration voltage of V_C volts to the input of the V/F converter; let N_C be the counts in τ seconds of the output. For a number of counts in the nth interval of τ for an arbitrary input of $V_i(t)$ defined as $N_{\tau n}$, the following expression defines $V_{\tau n}$:

$$V_{\tau_n} = \frac{N_{\tau_n} - N_o}{N_c - N_o} V_c$$
 (1)

But, because the V/F data recovery technique is an integrating method,

$$N_{r_n} - N_o = K_v \int_{n_r}^{(n+1)r} V_i(t) dt$$
 (2a)

and

$$N_{c.} - N_{o} = \int_{0}^{\tau} K_{v} V_{c} dt = K_{v} V_{c} \tau$$
 (2b)

where K_V is the gain of V/F converter with units of pulses per second-volt.

Substituting the definitions of Eqs. (2a) and (2b) into Eq. (1), it becomes:

$$V_{r_n} = \frac{1}{r} \int_{r_n}^{r_n} V_i(t) dt$$
 (3)

Equation (3) will be used to theoretically simulate the V/F conversion method of data acquisition and analyze the errors resulting from taking the value of V_{τ_n} as a data point that occurs at the midpoint of the summing interval from $n\tau$ to $(n+1)\tau$ and the points linearly connected to represent the input waveform. Figure 2 illustrates this type of data acquisition and waveform reconstruction.

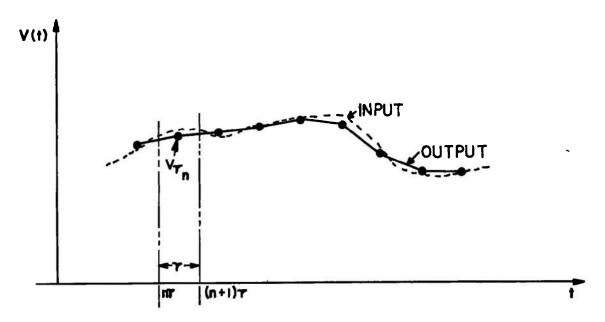


Fig. 2 Example Illustrating the Summing Interval Method of Data Representation

Since the value V_{τ_n} is an integral, its value and the value of $V_i(t)$ at $t = \tau_n = \frac{2n+1}{2}\tau$, the time at the midpoint of the nth interval, are not always equal; therefore, the error analysis also investigates the data point error. To distinguish the two, the lower case letter, e, designates the error from linear interpolation and the upper case letter, E, is used to designate the error at data points.

SECTION III SINUSOIDAL INPUT

For the sinusoidal analysis, let $V_i(t) = \sin \frac{2\pi t}{T}$, which is normalized for ease of application and does not affect the actual error analysis.

Substituting this function in Eq. (3), the expression for the representative output becomes:

$$V_{\tau_{n}} = \frac{1}{r} \int_{n\tau/}^{(n+1)\tau} \frac{2\pi t}{T} dt$$

$$= -\frac{T}{2\pi r} \left[\cos \frac{2\pi t}{T} \right]_{n\tau}^{(n+1)\tau}$$

$$= -\frac{T}{2\pi r} \left[\cos \frac{2\pi (n+1)\tau}{T} - \cos \frac{2\pi n\tau}{T} \right]$$
(4)

Using the identity from Ref. 3, page 18 for the difference of two cosine functions, Eq. (4) becomes:

$$V_{r_n} = \frac{T}{\pi r} \left[\sin \frac{(2n+1)\pi r}{T} \cdot \sin \frac{\pi r}{T_c} \right]$$
 (3)

Letting the ratio T/τ , which is the number of summing intervals per period, be designated with the letter m, the expression in Eq. (5) becomes:

$$V_{\tau_n} = \frac{m}{\pi} \left[\sin \frac{(2n+1)\pi}{m} \cdot \sin \frac{\pi}{m} \right]$$
 (6)

3.1 LINEAR INTERPOLATION ERROR ANALYSIS OF SINUSOIDAL INPUT

An expression for e consists of n equations; therefore, the determination of a maximum value is a tremendous task to perform manually. Consequently, a digital computer program was written to determine the errors for a sinusoidal input as m is varied from 1 to 25.0 in increments of 0.1. The program was designed to search the number of cycles that include a whole number of summing intervals. The reason for this approach to the e_{max} determination is that, after this number of cycles, the errors become repetitive. By incorporating this test, the e_{max} determination is expedited. The program is listed in Table I-I (Appendix).

The solutions for e_{max} versus m are plotted in Fig. 3. The plot depicts an irregular relationship, and the most significant fluctuations are in the range of m = 1.0 to 3.0. Therefore, little faith can be placed in the application of the basic sampling theorem of two summing intervals per period to the V/F system. Another significant factor is that, when the sampling rate is close to frequency components of the data that have prominent magnitude, the errors greatly reduce the accuracy of the data recovery.

As an example of the use of Fig. 3, consider an input voltage signal that contains a 50-Hz component. What error would be introduced in the data reduction of this component if τ were 10 msec? What is the error for τ = 2 msec? For τ = 10 msec,

$$m = \frac{10^3}{(50)(10)} = 2$$

From Fig. 3, emax = 44 percent.

For $\tau = 2$ msec,

$$m = \frac{10^3}{(50)(2)} = 10$$

From Fig. 3, e_{max} = 6 percent.

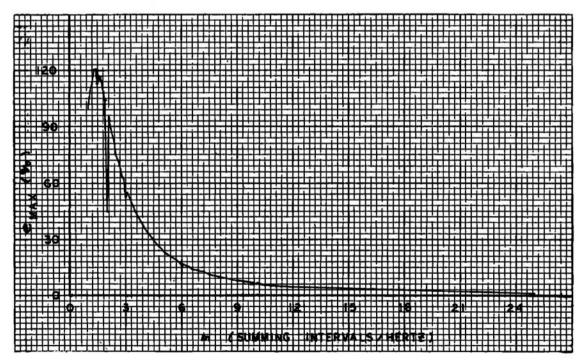
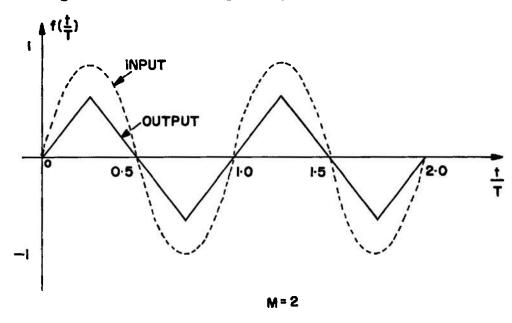


Fig. 3 Plot of m versus e_{max} for Sinusoidal Input

Figure 4 depicts the input-output relationship for these two summing intervals for the 50-Hz component.

In order that e_{max} be 5 percent or less for a sinusoidal input, m must be greater than 11.4 and at m = 2, e_{max} = 46 percent. This indicates a large error at the rate given by the sampling theorem.



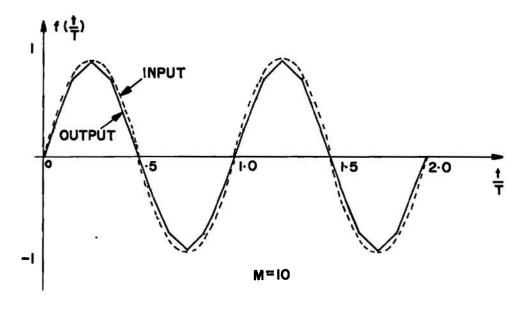


Fig. 4 Input-Output Waveforms for Sample Problem

3.2 DATA POINT ERROR ANALYSIS OF SINUSOIDAL INPUT

The error analysis for $E_{\mbox{max}}$ is much simpler than for $e_{\mbox{max}}$. The data point error is defined by this relationship:

$$E = 100 |V_i(r_n) - V_{r_n}|$$
 (7)

since $|V_i(t)|_{max}$ is equal to one.

Combining Eqs. (6) and (7) and substituting the normalized sinusoid with the definition of m included, the expression for E becomes:

E = 100
$$\left| \sin \frac{(2n+1)\pi}{m} (1 - \frac{m}{\pi} \sin \frac{\pi}{m}) \right|$$
 (8)

Even though the error is expressible in one equation, many calculations are required to find E_{\max} . Therefore, the program used to find e_{\max} also has an E_{\max} option. These solutions are depicted in Fig. 5.

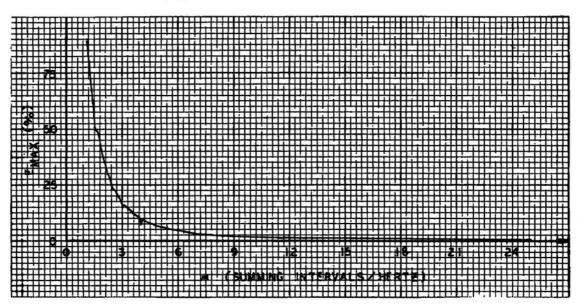


Fig. 5 Plot of m versus E_{max} for Sinusoidal Input

This plot of E_{max} versus m is a smoother curve than the one for e_{max} versus m. The minimum value of m for all data points to be within 5 percent is 5.7, and at the basic sampling rate given by the sampling theorem (two summing intervals/period), the data point is in error at a maximum by 36^+ percent.

3.3 SYNCHRONIZATION EFFECTS ON ERRORS OF SINSUOIDAL INPUT

The analyses of the sinusoid for e_{max} and E_{max} have made the assumption that the sinusoid and the n = 0 interval both started at a time when the value of the sinusoid was zero and had a positive slope. This section deals with the effects of synchronization upon e_{max} and E_{max} .

The input signal must be expressed with a synchronization parameter, ψ . The input then is:

$$V_1(t) = \sin\left(\frac{2\pi t}{T} + \psi\right)$$

Substituting this expression for V;(t) into Eq. (3):

$$V_{T_{n}} = \frac{1}{\tau} \int_{n\tau}^{(n+1)\tau} \sin\left[\frac{2\pi}{\tau}t + \psi\right] dt$$

$$= -\frac{T}{2\pi\tau} \cos\left[\frac{2\pi}{T}t + \psi\right]_{n\tau}^{(n+1)\tau}$$

$$= -\frac{T}{2\pi\tau} \cos\left[\frac{2\pi(n+1)\tau}{T} + \psi\right] - \cos\left[\frac{2\pi n\tau}{T} + \psi\right]$$

$$= \frac{T}{\pi\tau} \sin\left[(2n+1)\frac{\pi\tau}{T} + \psi\right] \cdot \sin\frac{\pi\tau}{T}$$
(9)

Using the definition of m, Eq. (9) becomes:

$$V_{\tau_n} = \frac{m}{\pi} \sin \left[\frac{(2n+1)\pi}{m} + \psi \right] \cdot \sin \frac{\pi}{m}$$
 (10)

Since many solutions of Eq. (10) are required to determine the synchronization effects, a program was written to vary m from 1 to 25.0 in increments of 0.5 and to vary ψ from 0 to 180 deg in increments of 1 deg. The solutions for the E_{max} option are in Table I and the solutions for the e_{max} option are in Table II. The program is listed in Table I-II (Appendix).

	DATA PO	INT ERROR	ANALYSIS		
		LARGEST		SMALLEST	DELTA
M	PS I	'E(MAX)	PSI	F(MAX)	E(MAX)
1.00	90.30	100.00	0.0_	0_• 0 0	100,00
1.50	30.00	58.65	0.0	50.79	7.86
2.00	0.0	36.34	9C.00	0.00	36.34
2.5C	18.00	24.32	72.00	23.13	1.19
3.00	30.00	17.30	C.O	1,4.98	2 . 32
3.50	90.00	12.90	0.0	12.57	0.32
4.0C	45.00	9.97	0.0	7.05	2.92
4.50	10.00	7.93	0.0	7.81	0.12
5•0€	18.00	6.45	0.0	6.14	0.32
5.50	41.00	5.35	0.0	5.30	0.05
6.00	0.0	4.51	30.00	3.90	0.60
6.50	90.00	3.85	0.0	3.82	0.03
7.00	90.00	3.32	0.0	3.24	0.78
7.50	6.00	2.90	60.00	2.89	0.02
8 • 00	22.00	2.55	90.00	2.36	0.19
8.50	16.00	2.26	180.00	2.25	0.01
9.00	10.00	_2.02		1.99	0.03
9.5C	52.00	1.81	0.0	1.81	0.01
10.00	0.0	1.64	18.00	1.56	0.08
19.50	73.00	1.49	120.00	1.48	0.20
11.60	8.00		180.00	1.34	2.01
11.50	43.00	1.24	0.0	1.24	0.00 0.04
12.00	15.00	_1.14		1.10 1.05	0.00
12.50 13.00	11.00 76.00	1.05	0.0	0.96	0.01
13.50	10.00	0.90	0.0	0.90	0.00
14.00	10.00	0.90	96.00	0.82	0.02
14.50	34.00	0.78	0.0	0.78	0.00
15.00	6.00	0.73		0.73	0.00
15.50	32.00	0.68	180.00	0.68	0.00
16.00	11.00		45.00	0.63	0.01
16.50	101.00	0.60	0.0	0.60	0.00
17.00 _	16.00_	<u>9.57</u>	0.0	0.57	0.00
17.50	85.00	0.54	108.00	0.54	0.00
18.00	G.C	<u>0.51</u>	1.0, 00	0.50	0.01
18.50	17.00	0.48	107.00	0.48	0.00
<u>19.00</u>	71.00	0.46	180.00	0.45	_ 0.00
19.50	67.00	0.43	60.00	0.43	0.00
20,00	9.00		0.0	0.41	0.01
20.50	11.00	. 0.39	180.00	0.39	0.20
. 21.00	4.00	0.37	180.00	0.37	000
21.50	23.C0	0.36	21.00	0.36	0.00
22.00	33.00 _	.0.34	90.00	0.34	0.00
22.50	2.00	0.32	12.00	0.32	0.00
23.00	59.00	0,31	0.0	0 <u>.31</u>	0.00
23.50	6.00	0.30 0.29	46.00	0.30 0.28	0.00 0.00
24.00 24.50	8,00 46.00	0.27	180.00 136.00	0.27	0.00
25.00	4.00	0.26	108.00	0.26	0.00
بان و دعر	7107		100.00		7,000

 $\begin{array}{c} \text{TABLEII} \\ \text{e_{mox} SOLUTIONS OF SYNCHRONIZATION EFFECTS ON SINUSOIDAL INPUTS} \end{array}$

M						
M			LADGEST		CMALLEST	DEL TA
1.00	M	05.1		DCf		
1.50						
2.00 49.00 100.00 76.61 C.2 73.75 2.85 3.00 90.00 76.61 C.2 73.75 2.85 3.00 90.00 58.65 0.0 52.15 6.50 3.50 90.00 45.69 0.0 44.68 1.02 4.50 90.00 29.47 C.0 29.23 0.44 5.50 90.00 24.32 36.00 23.18 1.14 5.50 90.00 17.30 0.0 15.06 2.24 6.00 90.00 17.30 0.0 15.06 2.24 6.50 90.00 14.86 55.00 14.74 0.12 7.00 90.00 12.90 0.0 11.25 0.33 7.50 90.00 12.90 0.0 11.20 0.10 8.50 90.00 18.6 0.0 9.24 0.73 8.50 90.00 18.6 0.0 7.79 0.14 9.50						
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	25.00	11.000	1.57	100.00	1.03	C • 90

These solutions depict the cases in which the variations of E_{max} or e_{max} are the largest. The cases are the integer values of m and m with a decimal part of 0.5. All other values of m exhibit less variation in E_{max} or e_{max} as ψ varies.

From Table I,

for m > 6, $\Delta E_{max} \le 0.19$ percent for m > 11.4, $\Delta E_{max} \le 0.04$ percent for m ≥ 14.0 , $\Delta E_{max} \le 0.01$ percent

The m > 11.4 values yield accurate recovery.

From Table II,

for m > 8, $\Delta e_{max} \le 0.32$ percent for m > 11.4, $\Delta e_{max} \le 0.19$ percent for m > 16, $\Delta e_{max} \le 0.05$ percent

This clearly indicates that synchronization has negligible effect on the error at a summing interval-to-period ratio that is required to accurately recover the data.

3.4 EXPERIMENTAL CORRELATION OF THEORETICAL ERRORS

A verification, by actual errors encountered in the data recovery, of the theoretical treatment was desired; therefore, an experiment to determine actual errors was formulated. For the input waveform, a sinusoid was chosen. There were, basically, two reasons for this choice: (1) a sinusoid is easily obtainable from a sinusoidal generator, and (2) the frequency or period of cycle is a very important factor in the error of a signal.

The VIDAR Data Acquisition System was set up with a sinusoidal generator connected to the input of a channel of the V/F converter. The frequency output was recorded, using direct electronics, on a magnetic tape recorder. The recorded tape was translated using a summing interval of 2 msec, and the raw counts were printed out.

In order to get the negative values of the sinusoid, the VIDAR was adjusted so that for zero input, the output counts were at midscale.

Then a known voltage of 1.5 v was applied, and the VIDAR spanned to nearly full scale. The count levels of these adjustments were:

0v Input = 25,003 counts/sec
1.5v Input = 47,500 counts/sec

The signal from the sinusoid generator was set at 20 Hz and 2 v peak-to-peak which is easily accommodated in magnitude by the chosen calibration voltage.

Since the theoretical error analysis used m as the variable, m had to be determined for this case of the translation summing interval; m was defined as T/τ or $1/f\tau$; giving m = 1/20 (0.002) = 25.

Synchronization of output and input for actual error analysis proved to be a difficult problem; therefore, since for an m of 25, e_{max} = 1.05 percent and E_{max} = 0.26 percent, this linear interpolated curve was used as the reference or standard. Small m's were obtained by taking two or more adjacent summing intervals at a time. A computer program, depicted in Table I-III (Appendix), was written to do the task of summing up adjacent summing intervals. The program limits the number of adjacent summing intervals to 13 since this gives an m of 1.92, which is below the level set forth by the sampling theorem. Table III is the solution for this program which has the option of determining either e_{max} or E_{max} .

As shown in Table III, the maximum errors determined by the theoretical approach were confirmed to be reasonably accurate; therefore, the general rule of thumb of 11.4 summing intervals per period for $e_{\mbox{max}} \leq 5$ percent is a valid one. This also indicates that the theoretical simulation is a reasonably accurate error determination technique.

SECTION IV

In the theoretical analysis of errors of the V/F converter data acquisition system in following a step input, the time reference of t=0 is set at the start of the summing interval in which the step function occurs.

TABLE III $\bullet_{max} \text{ and } E_{max} \text{ THEORETICAL AND ACTUAL SOLUTIONS FOR SINUSOIDAL INPUT }$

	ACTUAL	THEORY			ACTUAL .	THEORY
- 11	E(MAX)	E(MAX)	•	M N	E(HAX)	E(NAX)
1.92	105.31	103.68		2.08	94.82	05.63
2.27	35.46	ö ú.51		2.50	75.63	73.76
2.73	64.50	65.81		3.13	54,01	
3.57	43.32	44.08	19 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	4.17	`33.7ü	33 .7 5
5.00	23.88	23.18		6.25	16.34	15.93
3.33	10.47	9.19		12.50	5.35	4.15
DATA PÕIN	IT ERROR × ACTUAL E(MAX)	ANALYSIS THEORY E(MAX)	요한 #한 # # # # # # # # # # # # # # # # #		ACTUAL ? E(MAX) >	THEORY (
1.92	38.67	38.83		2.08		33.75
2.27	23.57	23.88		2.50		
2.78		19.96	The state of		15.35	
	- 13.47 - 8€00		San Barrell	- 14.4/ ? - 13版 - 13E	• 8.57 * b. 200	9.19
8.33	4.76	2.35	7 3 34 57	∵∯ vũ.25 v 12.50	0.01	1 00
0.73	4.70	4.77		12.50	0.01	1.05

The input waveform, a normalized step function, is denoted by the symbol:

$$\mu_1 \quad (t-a) = \begin{cases} 0 & t < \alpha \\ 1 & t \ge \alpha \end{cases} \tag{11}$$

For this analysis, the input is:

$$V_i(t) = \mu_i (t - Kr)$$
 (12)

where $0 \le K \le 1$ is the shift of the step with respect to the summing interval reference.

Using Eqs. (3) and (12) to find the output, the particular interval for n = 0 is the only one of interest since in all intervals for n < 0, $V_{\tau_n} = 0$, and in all intervals for n > 0, $V_{\tau_n} = 1$.

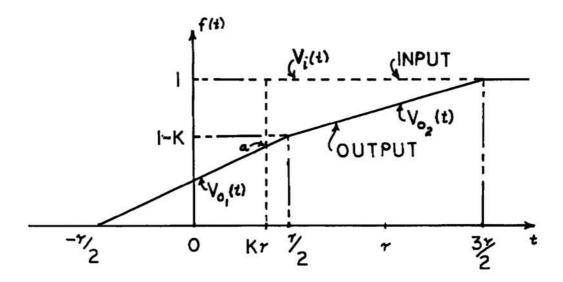
The interval for n = 0 has an output value of:

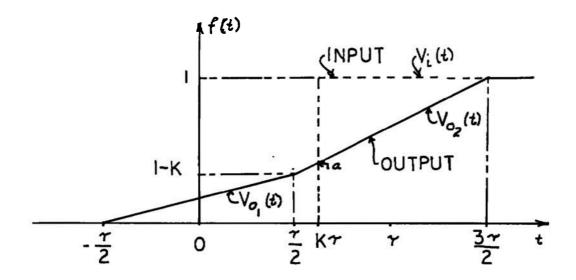
$$V_{r_0} = \frac{1}{r} \int_0^r \mu_1 \ (t - Kr) dt$$

$$= \frac{1}{r} \int_{Kr}^r dt = \frac{1}{r} (r - K)$$

$$= 1 - K$$
(13)

The input-output relationship is depicted in Fig. 6.





1/2 ≤ K ≤ 1

Fig. 6 Input-Output Waveforms for Step Input

4.1 LINEAR INTERPOLATION ERROR ANALYSIS OF STEP INPUT

As shown in Fig. 6, the maximum error occurs at $t = K\tau$. Point a is used to designate this intersection of the input and output waveforms at $t = K\tau$. The output line from $t = -\frac{\tau}{2}$ to $\frac{\tau}{2}$ is designated V_{01} (t), and the output line from $t = \frac{\tau}{2}$ to $\frac{3\tau}{2}$ is designated V_{02} (t). For $0 \le K \le 1/2$, point a occurs on the output line V_{01} (t). The equation for this line, which has a slope of $\frac{1}{\tau}$ (1-K) and passes through the point $(-\frac{\tau}{2}, 0)$, is:

$$V_{0_1}(t) = \frac{1}{r}(1 - K)(t + \frac{r}{2})$$

Since $|V_1(t)|_{max} = 1$,

$$e_{max} = 100 V_{o_1}(K_r)$$

$$= 100 (1 - K) (K + \frac{1}{2})$$

$$= 50 (1 - K - 2K^2)$$
(14)

for $0 \le K \le 1/2$.

For $1/2 \le K \le 1$, point a occurs on the output line V_{02} (t). The equation for this line, which has a slope of K/τ and passes through the point $(\frac{3\tau}{2}, 1)$, is:

$$V_{o_2}(t) = \frac{K}{\tau} (t - \frac{3\tau}{2}) + 1$$

Likewise,

$$e_{max} = 100 [1 - V_{o_2} (K_7)]$$

= -100K (K - 3/2)
= 150K - 100K²
= 50 (3K - 2K²) (15)

for $1/2 \le K \le 1$.

Figure 7 is a plot of e_{max} versus K using Eqs. (14) and (15). As can be seen, e_{max} varies from the lowest value of 50 percent at K = 0, 1/2, and 1 to a highest value of 56.25 percent at K = 1/4 and 3/4.

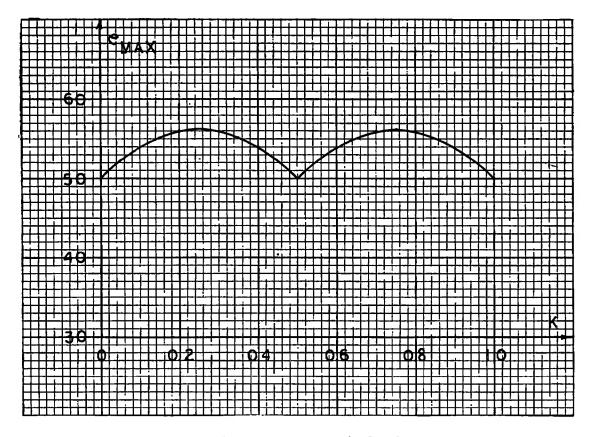


Fig. 7 Plot of K versus e_{max} for Step Input

4.2 DATA POINT ERROR ANALYSIS OF STEP INPUT

For this input signal, all data point errors are 0 percent with the exception of the one for n=0. This data point of interest occurs at time $t=\tau_0$ or 0.5.

Using Eq. (13) and the fact that $|v_i(t)|_{max} = 1$, the E_{max} for step input is:

$$E_{max} = 100 |V_i(r_0) - (1 - K)|$$
 (16)

However, $V_i(\tau_0)$ is dependent on the value of K in the following manner:

$$V_{i}(r_{o}) = \begin{cases} 1 & 0 \leq K \leq 0.5 \\ 0 & 0.5 \leq K \leq 1.0 \end{cases}$$
 (17)

Combining Eqs. (16) and (17), the maximum error becomes:

$$E_{max} = \begin{cases} 100K & 0 \le K \le 0.5 \\ 100 & (1 - K) & 0.5 \le K \le 1.0 \end{cases}$$

A plot of E_{max} versus K is shown in Fig. 8. As was the case in the total error, the maximum data point error is independent of the summing interval length and is a function of K alone.

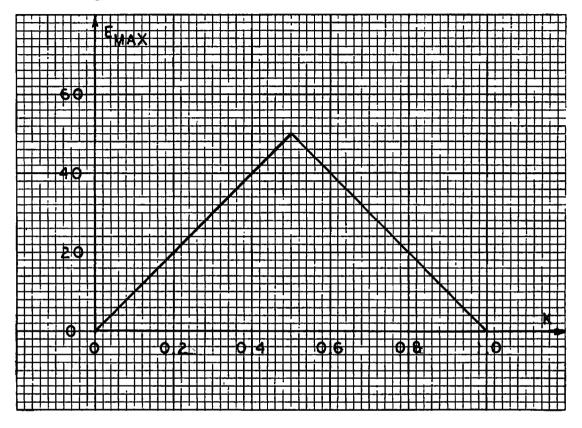


Fig. 8 Plot of K versus Emax for Step Input

4.3 TIME DELAY ANALYSIS OF STEP INPUT

The determination of time delay, i.e., time for the response of the linear interpolated output to be within 5 percent error, was desired. The notation T_D is used for this parameter.

This determination must, like the error analysis, be divided into two parts. Likewise, the first part uses the output, V_{01} (t), in its determination. The ranges of K, however, are not the same. In using V_{01} (t), the following inequality must be satisfied:

$$1 - V_{o_1}(\frac{t}{2}) \le 0.05$$
 (18)

Since $V_{01}(\frac{7}{2}) = 1-K$, Eq. (18) defines the limits of K to be:

$$0 \le K \le 0.05$$

For a time delay of T_D , the actual time $t = T_D + K\tau$, and from the equation for V_{O1} (t),

$$V_{o_1}(T_D + K_r) = 0.95 = \frac{1}{r}(1 - K)(T_D + K_r + \frac{r}{2})$$

Solving for TD,

$$T_{\rm D} = \frac{0.45 - 0.5K + K^2}{1 - K} \tag{19}$$

for $0 \le K \le 0.05$.

The second part, $0.05 \le K \le 1$, uses V_{02} (t) in the determination of T_D , with $t = T_D + K\tau$.

$$V_{o_2}(T_D + K_r) = 0.95 = \frac{K}{r}(T_D + K_r - 1.5r) + 1$$

Solving for TD,

$$T_{\rm D} = \frac{\sqrt{0.05 + 1.5K - K^2}}{K^2}$$
 (20)

for $0.05 \le K \le 1$.

Using Eqs. (19) and (20), a plot of T_D versus K is depicted in Fig. 9. As shown in this plot, T_D varies from a minimum of 0.4497 to a maximum of 1.0537.

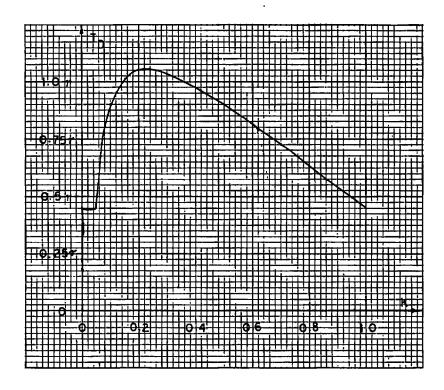


Fig. 9 Plot of K versus TD for Step Input

The step response will always appear to be a ramp. For K = 1/2, the rise time of the ramp is 2τ sec, and for K = 0 or 1, the rise time will be τ sec; therefore, to minimize this effect, τ must be made as small as possible. Also, the output will respond to within 5 percent error in a delay time, T_D , of approximately τ .

SECTION V RAMP INPUT

In order to theoretically analyze the error of the V/F converter data acquisition system in the data recovery of a ramp input, time, t=0, is set at the start of the summing interval in which the ramp function begins.

The ramp function is defined as:

$$\mu_2 (t - a) = \begin{cases} 0 & t < a \\ t - a & t > a \end{cases}$$
 (21)

For a normalized ramp function which has a rise time of β summing intervals, the input function is expressed as:

$$V_{i}(t) = \frac{1}{\beta \tau} \left[\mu_{2} (t - K\tau) - \mu_{2} (t - (K + \beta) \tau) \right]$$
 (22)

The value of V_{τ_O} is dependent upon the value of $K + \beta$. For $(K + \beta) \le 1$, the combination of Eqs. (3) and (22) becomes:

$$V_{\tau_{0}} = \frac{1}{\beta r^{2}} \int_{0}^{\tau} \left[\mu_{2} \left(t - Kr \right) - \mu_{2} \left(t - (K + \beta) r \right) \right] dt$$

$$= \frac{1}{\beta r^{2}} \int_{Kr}^{(K + \beta)r} \left(t - Kr \right) dt + \frac{1}{\tau} \int_{[(K + \beta)r]}^{\tau} dt$$

$$= \frac{1}{\beta r^{2}} \left[\frac{1}{2} t^{2} - Kr t \right]_{Kr}^{(K + \beta)r} + \frac{1}{\tau^{2}} \left[t \right]_{(K + \beta)r}^{\tau}$$

$$= \frac{1}{\beta r^{2}} \left[\frac{1}{2} (K + \beta)^{2} r^{2} - \frac{1}{2} K^{2} r^{2} - K (K + \beta) r^{2} + K^{2} r^{2} \right]$$

$$+ \frac{1}{\tau} \left[r - (K + \beta) r \right]$$

$$= \frac{\beta}{2} + 1 - (K - \beta)$$

$$= 1 - K - \frac{\beta}{2}$$
(23)

for $0 \le K + \beta \le 1$.

For $(K + \beta) > 1$, V_{τ_0} is defined by the integral:

$$V_{\tau_{0}} = \frac{1}{\beta r^{2}} \int_{K\tau}^{\tau} (t - Kr) dt$$

$$= \frac{1}{\beta r^{2}} [\frac{1}{2}t^{2} - Kr]_{K\tau}^{\tau} = \frac{1}{\beta r^{2}} [\frac{1}{2}t^{2} - K + K^{2}]$$

$$= \frac{(1 - K)^{2}}{2\beta}$$
(24)

for $(K + \beta) > 1$.

For all summing intervals up to, but not including, the one in which the ramp breaks to a zero slope, the value of the output is equal to the value of the input at the midpoint of the summing interval. That is:

$$V_{r_n} = V_i \left(\frac{2n+1}{2} r \right)$$

for $0 \le n \le Int (K + \beta)$.

For the interval $n = Int (K + \beta)$, the output value is:

$$V_{r_{n}} = \frac{1}{\beta r^{2}} \int_{n\tau}^{(K+\beta)\tau} (t - K\tau) dt + \frac{1}{\tau} \int_{(K+\beta)\tau}^{(n-1)\tau} dt$$

$$= \frac{1}{\beta r^{2}} \left[\frac{1}{2} t^{2} - K\tau \right]_{n\tau}^{(K+\beta)\tau} + \frac{1}{\tau} \left[t \right]_{(K+\beta)\tau}^{(n+1)\tau}$$

$$= \frac{1}{2} (K + \beta)^{2} - \frac{K (K+\beta)}{\beta} - \frac{n^{2}}{2\beta} + \frac{Kn}{\beta} + (n + 1) - (K + \beta)$$

$$= (n + 1) - K - \beta/2 - \frac{(n - K)^{2}}{2\beta}$$
(25)

Figure 10 depicts the input-output relationships for a ramp function.

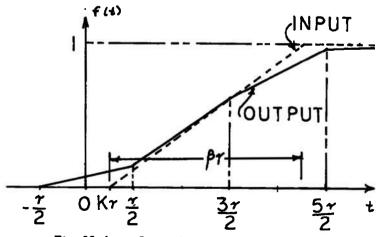


Fig. 10 Input-Output Waveforms for Ramp Input

5.1 LINEAR INTERPOLATION ERROR ANALYSIS OF RAMP INPUT

As shown in Fig. 10, the e_{max} occurs at either $t = K\tau$ or $t = (K + \beta)\tau$; therefore, e_{max} is equal to the maximum of these two values. An expression for the e_{max} is so cumbersome to deal with manually that a digital computer program, depicted in Table I-IV (Appendix), was written to calculate the many solutions required to determine the minimum β for 5 percent or less error. The program calculates the β_{min} for $e_{max} \leq 5$ percent; then β is varied from 0.1 to β_{min} in increments of 0.1 for K = 0.0 and 0.50 and the value of e_{max} determined for each set of parameters. The family of curves that was calculated is depicted in Fig. 11. These two values of K are the extreme cases; all other values are in between. The plots in Fig. 11 indicate that a value of $\beta \geq 5.0$ assures a response within 5 percent.

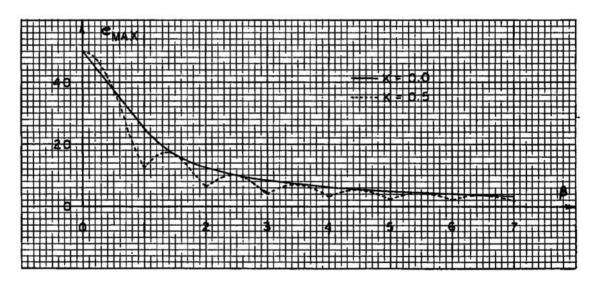


Fig. 11 Plot of e_{max} versus β for Ramp Input with K Constant

5.2 DATA POINT ERROR ANALYSIS OF RAMP INPUT

The value of E_{max} , as shown in Fig. 10, occurs at either $t_1 = 1/2\tau$ or $t_2 = [Int (K + \beta) + 1/2] \tau$. The expression for E_{max} is the maximum of:

$$E_1 = 100 |V_i|(\frac{1}{2}r) - V_{r_0}|$$

 \mathbf{or}

$$E_2 = 100 \ V_i \left(\frac{2n+1}{2}r\right) - V_{r_n}$$

where n = Int $(K + \beta)$.

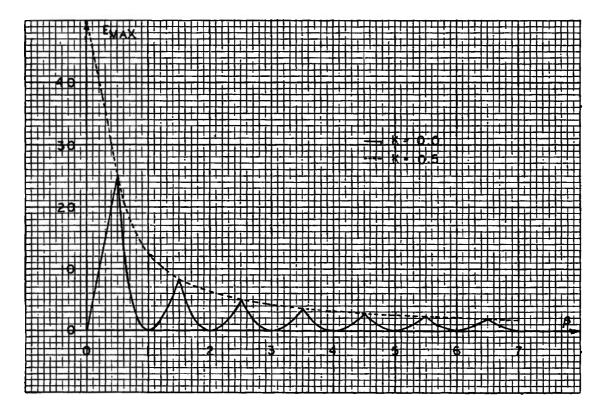


Fig. 12 Plot of E_{max} versus β for Ramp Input with K Constant

The digital computer program that calculates e_{max} was written for the E_{max} option also. The value of β_{min} for $E_{max} \leq 5$ percent is first calculated; then β is varied from 0.1 to β_{min} in increments of 0.1 for K=0.0 and 0.5, since these errors are the extreme cases, and the value of E_{max} determined for each set of parameters. Figure 12 depicts the family of curves that was calculated. These plots indicate that, for a value of $\beta \geq 2.5$, the data points are in error by 5 percent or less. To illustrate, for a summing interval of 2 msec, any ramp with a rise time of 5 msec or greater will be reconstructed with all data points within 5 percent error.

SECTION VI SUMMARY AND CONCLUSIONS

6.1 SUMMARY

A voltage-to-frequency data acquisition system was theoretically analyzed for errors that occur when it is used as a sampling system. Since the standard sampling theorem is not directly applicable, the system was simulated on a digital computer to aid in the determination of parameter criteria for expected levels of accuracy for sinusoidal, step, and ramp inputs.

Parameter limits were determined for such parameters as m, summing intervals per period of a sinusoid, K, delay of step and ramp inputs with respect to start of summing interval, and β , ratio of ramp rise time to summing interval length.

An investigation of the effect of signal synchronization was accomplished for the sinusoid, and an experiment was performed, using a sinusoid, to determine the validity of the theoretical solutions.

6.2 CONCLUSIONS

The voltage-to-frequency conversion for data acquisition does not offer accurate recovery (5 percent error) of sinusoidal data until the ratio of summing interval to period is about 11.4, i.e., 11.4 summing intervals per period. This means that for a 2-msec summing interval, any frequency component above 43.9 Hz has a maximum error greater than 5 percent and, from Fig. 3, any component of 151.5 Hz or greater is in error at its maximum by more than 50 percent.

The voltage-to-frequency conversion system has an inherent error because all data points do not coincide with the input waveform. For the sinusoid, the ratio of summing intervals to period must be 12.8 for the data point to be within 1 percent of the input waveform. At the value of m = 11.4, e_{max} was 4.95 percent and E_{max} is 1.26 percent; therefore, a sizeable portion of the error is the result of the data point error.

The step input error is independent of the summing interval and is symmetrical with respect to K = 0.5 for both e_{max} and E_{max} . However, delay time, time required to bring the output to be within 5 percent of the input, is a function of K and τ , and as K varies, the time delay varies in the range from 0.45 τ to 1.05 τ .

Ramp inputs can be followed within 5 percent if the ratio of ramp rise time to summing interval is larger than five. This is to say that a 2-msec summing interval can accurately follow a ramp of rise time greater than 10 msec.

The ramp function has an unusual E_{max} versus β for constant K relationship. At low and high values of K, the E_{max} oscillates as a function of β . At midvalues of K, the relationship is nearly exponential.

If $\beta \ge 12.5$, all data points are within 1 percent of the input waveform. The reverse conditions exist in the relationship of e_{max} versus β for constant K.

An additional error that may be significant at small summing interval lengths is resolution error from the low pulse counts that occur. However, errors from resolution were not included in this analysis.

REFERENCES

- Mallinckrodt, A. J. "Aliasing Errors in Sampled Data Systems." Report 316 AGARD, NATO, Paris, 1961.
- 2. Tucker, G. K., and Willis, D. M. A Simplified Technique of Control System Engineering, Minneapolis-Honeywell Regulator Company, Brown Instrument Division, Philadelphia, 1962.
- 3. Burington, R. S., Compiler. Handbook of Mathematical Tables and Formulas. Handbook Publishers, Inc., Sandusky, Ohio, 1954.

APPENDIX COMPUTER PROGRAMS

TABLE I-I

COMPUTER PROGRAM TO FIND E_{max} AND e_{max} FOR A SINUSOIDAL INPUT

AS m VARIES FROM 1.0 TO 25.0 IN INCREMENTS OF:0.1

L.0001	/JOB	GO, TIME=10
L.0002		DIMENSION XMA(241), DIFMA(241)
L.0003	C	IND=0 (E(MAX) FOR DATA POINT)
L.0004	С	IIID=1 (E(MAX) FOR LINEAR INTERPOLATION AT ANY POINT)
L.0005		READ(5,1) IND
L.0006	1	FORMAT(110)
L.0007		PSI=0.
L.0008		PSI=0. DO 103 IM=10,250
L.0009		1=IH-9
L.0010		Xf1=FLOAT(If1)/10.
L.0011		XIIA(I)=XI1
L.0012		DIFMA(I)=VEMAX(XM, PSI, IMD)
L.0013	103	CONTINUE
L.0014		WRITE(6,2)
L.0015	2	FORMAT('1')
L.0016		IF(IND)100,100,101
L.0017	100	WRITE(6,3)
L.0018	3	FORMAT(10X'DATA POINT ERROR ANALYSIS'/)
L.0019	101	WRITE(6,4) (XMA(J),DIFMA(J),J=1,241)
L.0020	4	MRITE(6,4) (XMA(J),DIFMA(J),J=1,241) FORMAT(4(5X'M'4X'E(MAX),4X)/4(2F8,2,4X))
L.0021		STOP,
L.0022		END

TABLE I-I (Concluded)

```
L.0001
               FUNCTION VEHAX (XM, PSI, IND)
               DIMENSION FA(252), TA(252)
L.0002
L.0003
               PI=3.1415926
               PSI = PSI * PI / 180.
L.0004
L.0005
               IF(XI1-25.)99,99,98
L.0006
        98
               STOP 2525
L.0007
         99
               DO 110 L=10,100,10
L.0008
               XP=0.001
               DO 109 |=1,2
L.0009
L.0010
               IXC=XII*FLOAT(L)+XP
L.0011
               IF(IXC-(IXC/10)*10)103,103,110
L.0012
         103
               XP=XP+0.9
L.0013
         109
               CONTINUE
L.0014
               GO TO 111
L.0015
         110
               CONTINUE
L.0016
        111
               NS = (IXC + 25)/10
L.0017
               ARG1=PI/XN
L.0018
               EMAX=0.
L.0019
               DO 100 N=1,NS
               XN=N-2
L.0020
               ALPHA=2. *XN+1.
L.0021
               TA(II)=ALPIIA/XII/2.
L.0022
L.0023
               ARG2=ALPHA*ARG1+PSI
               FA(N)=SIN(ARG2)+SIN(ARG1)/ARG1
L.0024
L.0025
               ERR=ABS(SIN(ARG2)-FA(N))
L.0026
               EMAX=AMAX1(ERR, EMAX)
L.0027
         100
               CONTINUE
               IF(IND)103,103,101
L.0028
L.0029
         101
               EMAX = 0.
               IXS=TA(NS)*100.+1.
L.0030
L.0031
               DO 102 |X=1, |XS
L.0032
               X=FLOAT(IX)/100.
L.0033
               TEST=TAB2(FA,TA,X,NS)
               DEV=ABS(SIN(2.*PI*X+PSI)-TEST)
L.0034
L.0035
               CHAX = AMAX1 ( EMAX, DEV )
L.0036
         102
               CONTINUE
L.0037
         103
               VEMAX=EMAX*100.
               RETURN
L.0038
L.0039
               END.
```

TABLE 1-II

COMPUTER PROGRAM TO FIND THE LARGEST, SMALLEST, AND DELTA

Emax OR emax as \$\psi\$ VARIES FROM 0 TO 180 deg IN INCREMENTS OF

1 deg AND m VARIES FROM 1 TO 25 IN INCREMENTS OF 0.5

0001	DIMENSION TA(252), FA(252)
0002	PI=3.1415926
0003	DO 906 ND=1,2
0004	
0005	WRITE(6,1)
_0006	to the state of th
0007	IF(IND)902,902,903
	902_WRITE(6,2)
0009	2 FORMAT(//15X'DATA POINT ERROR ANALYSIS')
0010	
0011	3 FORMAT (/23x*LARGEST*13x*SMALLEST DELTA*/7X
	1'M'2X2(6X'PSI'5X'E(MAX)'),4X'E(MAX)')
0012	DO 906 IM=10,250,5
0013	
0014	DD 99 I=1,252
	99 TA(I)=(2.*FLOAT(I-2)+1.)/XM/2.
	EMAQUT=0.
0016	
	EMIOUT=150.
0018	00 102 L=10,100,10
	XP=0.001
0020	DD 101 I=1.2
	IXC=XM*FLOAT(L)+XP
0022	IF(IXC-{IXC/10)*10)100,100,102
0023	100 XP=XP+C.9
00 24	101 CONTINUE
	GO TO 103
0026	102 CONTINUE
,0027	103 NS=(IXC+25)/10
0028	ARG1=PI/XM
0029	00 900 I=1.181
00 30	CS [= 1-1
0031	PSI=CSI*PI/180.
0032	EMAX=0.
CO33	DD 104 N=1.NS
0034	ARG2=(2.*FLUAT(N-2)+1.)*ARG1+PSI
0035	FA(N)=SIN(ARG2)*SIN(ARG1)/ARG1
0036	ERR=ABS(SIN(ARG2)-FA(N))*100.
0037	EMAX=AMAXL(ERR+EMAX)
0038	104 CONTINUE
0039	IF(IND)107,107,105
0040	105 EMAX=0.
0041	
0042	K=2
0043	DO 106 IX=1.IXS
0044	X=FLUAT(IX-1)/100.
0045	701 DIF=X-TA(K)
0046	IF(DIF)703,703,702
0047	702 K=K+1
0048	GO TO 701
0049	703 DEV=ABS(SIN(2.*PI*X+PSI)-DIF*(FA(K)-FA(K-1))
	1/(TA(K)-TA(K-1))-FA(K))*100.

TABLE I-II (Concluded)

C051	106	CONT INUE
0052	107	IF(EMACUT-EMAX)8CO,801,801
00 53	800°	EMA:IUT=EMAX
CC 54		PSIL=CSI
OC 55	801	IF(FMAX-EMIUUT)802,900,900
20.56	802	EMIDUT=EMAX
0057		PSIS=CSI
0058	900	CONTINUE
0059		DIFM 4= EMADUT-EMIOUT
0060		WRITE(6,4) XM, PSIL, EMAOUT, PSIS, EMIOUT, DIFMA
0061	4	FORMAT(6F10.2)
0062	906	CONTINUE
0063		STOP
0064		END

TABLE I-III

COMPUTER PROGRAM TO COMPARE THEORETICAL AND ACTUAL

•mox or Emax FOR SINUSOIDAL INPUT

```
L.0001
        /JOB GO, TIME=10
L.0002
               DIMERSION DATA(292), FA(292), TA(292), FB(140), TB(140),
L.0003
              1XH(12), EHAK(12), ETHAX(12)
               READ(5,1) HBD, 120, HUC, VU, DATA
L.0004
               FORMAT(3/10,F10.3/(16F5.1))
L.0005
               WRITE(6,2)
FORMAT('1')
L.0006
L.0007
         2
               TAU=0.002
L.0008
               VKT=TAU*FLOAT(1UC-1ZC)/VU
E.0000
               ZF=FLOAT(1ZC) *TAU
L.0010
               V!1AX =0.
L.U011
               DO 100 N=1,292
L.0012
L.0013
               TA(11) = FLOAT(2*11-27)*TAU/2.
               FA(N) = (DATA(N) - ZF)/VKT
L.0014
L. UU15
        100
               VHAK=AHAX1(VHAX,ABS(FA(N)))
L. U016
               DO 104 J=2,13
               XJ=15-J
L.U017
L.0018
               LX*UAT=LUAT
L.0019
               VX*T3V=VXT*XJ
L.0020
               TZCJ=ZF*XJ
               KH(J-1)=0.05/TAUJ
L.0021
               ETHAX(J-1)=VEHAX(XH(J-1),0.,IND)
L.0022
L.0023
               Eill = 0.
               K = 279/(15-J)+1
L.0024
               DO 102 L=1, K
L.0025
L.0026
               D!1=0.
L.0027
               LL=1+(15-J)*(L-2)
L.0028
               LU=LL+14-J
L.0029
               00 101 I=LL, LU
               DH=BH+DATA(1)
L.0030
         101
L.0031
               FB(L)=(DK-TZCJ)/VKTJ
               TB(L) = FLOAT(2 + L - 3) + TAUJ/2.
L.0032
               VV1=TAG2(FA, TA, TB(L), 292)
L.0033
               ERR=AHAX1(ABS(VV1-FB(L)), ERR)
L.0034
               COUTTINUE
L.0035
         102
L.0036
                IF(IND)105,105,106
L.0037
         106
                ERR = 0.
               1)0 103 1=1,1001
L.0038
               THRE=FLOAT(1-1)*0.0005
L.0039
L.0040
               VV1=TAB2(FA,TA,THHE,292)
L.0041
               VV2=TAB2(FB,TB,TIME,K)
                ERR=AMAX1(ABS(VV1-VV2), ERR)
L.0042
         103
L.0043
         105
                EMAX(J-1)=ERR/VMAX*100.
L.0044
         104
                CONTINUE
L.0045
                IF(IMD)107,107,108
         107
L.0046
               WRITE(6,3)
                FORHAT(10X DATA POINT ERROR ANALYSIS!)
L.UU47
         3
         108
               WRITE(G, 4) (XH(J), EMAX(J), ETMAX(J), J=1,12)
L.0048
                FORMAT(2(20%'ACTUAL'2%'THEORY'4%)/2(15%'M'4%'E(MAX)'2%
L.0049
              1'E(MAX)'4X)//2(10X3F8.2,4X))
L.0050
L.0051
                STOP
L.0052
                END
```

TABLE I-IV COMPUTER PROGRAM TO FIND e_{max} OR E_{max} FOR A CONSTANT K AS β VARIES FROM 0.1 TO 12.0 IN INCREMENTS OF 0.1

L.0001	/J0B	60
L.0002		DIMENSION ERR(150), B(150)
L.0003	C	IND=0 (E(MAX) FOR DATA POINT)
L.0004	C	IND=1 (E(MAX) FOR LIBEAR INTERPOLATION AT ANY POINT)
L.0005	-	DO 105 J=1, 2
L.0006		IND=2-J
L.0007		NRITE(6,2)
	2	FORMAT('1')
L.0008	2	
L.0009	0.7	IF(IND)97,97,98
1.0010	97	IP=2101362752
L.0011		GO TO 99
L.0012	98	I P=1077952576
L.0013	99	NB=120
L.0014		DO 105 I=1,4.
L.0015		XK=FLOAT(I-1)/4.
L.0016		DO 104 B=1, HB
L.0017		B(B) = FLOAT(B) * 0.1
L.0018		ERR(1B) = EIIAXR(B(1B), XK, IND)
L.0019	104	CONTINUE
L.0020	T 0 7	WRITE(6,3) IP, XK, (B(IB), ERR(IB), IB=1, NB)
	7	FORMAT(//' ERROR ANALYSIS FOR BETA'A1' WITH K = 'F5.2
L.0021	3	1//6(4X'BETA E(MAX)'4X)//6(2F8.2,4X))
L.0022	705	
L.0023	105	CONTINUE
L.0024		STOP _
L.0025		END
L.0026		FUNCTION_ENAXR(BETA, XK, LND)
L.0027		DIMENSION FM(23), TN(23)
L.0028	C	IND=0 (E(MAX) FOR DATA POINT)
L.0029	C	IND=1 (E(MAX) FOR LINEAR INTERPOLATION AT ANY POINT)
L.0030		RAMPF(T) = AMM $M1(1., AMAX1(T-XK, 0.)/BEIA)$
L.0031		Fi(1) = 0.0
L.0032		TN(1)=-0.5
L.0033		DO 100 J=2,23
L.0034		TN(J)=TN(J-1)+1\$
L.0035	100	FN(J)=1.0
L.0036	100	TS1=XK+BETA
		IS1=TS1
L.0037		
L.0038		TS2=IS1
L.0039		S2= S1+1
L.0040		TS3=TS2-XK
L.0041		FN(2)=1XK-BETA/2.
L.0042		fF(S1-1)105,101,101
L.0043	101	FN(2)=(1XK)**2/BETA/2.
L.0044	_	IF(1S1-2)104,102,102
L.0045	102	NO 103 J=3,1S2
L.0046	103	FN(J)=RAMPF(TN(J))
L.0047	104	FN(1S2+1)=TS3+1(BETA+TS3**2/BETA)/2.
L.0048	105	IF(IND)106,106,107
		EI11=ABS(RAMPF(0.5)-FI.(2))
L.0049	106	
L.0050		EM2=ABS(RAMPE(TS2+0.5)-F!!(1S2+1))
L.0051	70-	GO TO 108
L.0052	107	EI11=ABS(TAB2(FN, TN, XK, 13))
L.0053		EM2=ABS(TAB2(FN,TN,TS1,13)-1.)
L.0054	108	EMAXR=AMAX1(EM1, EM2) *100.
L.0055		RETURN
1.0056		EIID
12502		

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13. ABSTRACT

The errors generated by the recovery of data acquired by a voltage-to-frequency (V/F) data acquisition system are theoretically analyzed, using a digital computer. The input waveforms considered are the sinusoid, the step function, and the ramp function. In each case, the well known "sampling theorem" of C. E. Shannon, viz, two samples per period of the smallest period present in the signal are sufficient for recovery of data with perfect fidelity, will be shown to be inapplicable as a criterion for accurate and reliable recovery of data acquired by a V/F system. The errors are of two categories: One is the error from linear interpolation recovery, and the other is from the fact that data points do not always coincide with the input.

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